

Austin DeLiccio

1. Data: The number of incidents of crime in NYC in 3 random weeks 2022 was on average 2000 per week with a sample standard deviation 42 and in 5 random weeks in 2021 was on average 1700 per week, with a sample standard deviation of 36.

Assuming that the number of incidents per week is a normally distributed random variable and that the variances are the same in the two years but unknown, test the null hypothesis the mean crime rate is unchanged against the alternative that it is higher in 2022, at 0.05 level of significance.

$H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 - \mu_2 < 0$

2021 μ_1 2022 μ_2

$$S_p^2 = \frac{4 \cdot 36^2}{6} + \frac{2}{6} \cdot 42^2 = 1452$$

$$S_p = \sqrt{1452}$$

$$T = \frac{-300}{\sqrt{1452} \sqrt{\frac{1}{5} + \frac{1}{3}}} = \frac{-300}{\sqrt{1452 \cdot \frac{8}{15}}} = -10.78$$

$$T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1 + n_2 - 2}$$

$-10.78 < -t_{.05, 6} = -1.943$

Reject the Null

2. In the previous question, test the null hypothesis that the variance in 2022 is 1849 against the alternative that it is not, at .05 level of significance.

$$\frac{(n-1)s^2}{\sigma_0^2} = \frac{2 \cdot 42^2}{1849} \sim \chi_2^2$$

$H_0: \sigma^2 = 1849$

$H_1: \sigma^2 \neq 1849$

Reject if $\frac{2 \cdot 42^2}{1849} > \chi_{.025, 2}^{2 \sim 7.38}$ or $\frac{2 \cdot 42^2}{1849} < \chi_{.975, 2}^{2 \sim 0.506}$

≈ 1.91 ≈ 1.91

Do not reject the Null

3. Continuing, test the null hypothesis that the variance in the two years are equal, against the alternative that it was higher in 2022, at 0.01 level of significance.

$$\frac{S_1^2}{S_2^2} \sim F_{n_1-1, n_2-1}$$

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 < \sigma_2^2$

Reject if $\frac{S_1^2}{S_2^2} \leq F_{.99, 4, 2}$